

# *HyperAgent*: A Simple, Efficient and Scalable RL framework in Complex Environment

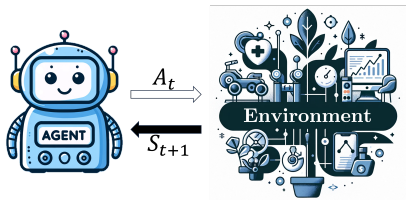
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# Reinforcement Learning Problem



**Figure:** Agent-Environment Interface.

Experience:  $A_0, S_1, A_1, S_2, \dots$

Environment  $M = (\mathcal{S}, \mathcal{A}, P)$

- ▶ State  $S_{t+1} \sim P(\cdot | S_t, A_t)$ .

Agent( $\mathcal{S}, \mathcal{A}, r, \mathcal{D}_t$ )  $\rightarrow \pi_t$  to max long-term rewards

- ▶ Reward  $R_{t+1} = r(S_t, A_t, S_{t+1})$  where  $r$  describes the Agent's preference.
- ▶ Historical **Data**  $\mathcal{D}_t = \mathcal{D}_{t-1} \cup \{A_{t-1}, S_t\}$  is **accumulated** with initial  $\mathcal{D}_0 = \{S_0\}$  or  $\mathcal{D}_0 = \mathcal{D}_{\text{offline}}$ .
- ▶ Action  $A_t \sim \pi_t(\cdot | S_t)$ ; Policy  $\pi_t = \text{Agent}(\mathcal{S}, \mathcal{A}, r, \mathcal{D}_t)$  adapted to the **accumulated**  $\mathcal{D}_t$ .
- ▶ **Objective:**  $\pi_{\text{agent}} = (\pi_0, \pi_1, \dots)$  to maximize

$$\mathbb{E} \left[ \sum_{t=0}^{T-1} R_{t+1} \mid \pi_{\text{agent}}, M \right]. \quad (1)$$

# Complex Environment has Exponentially Large $\mathcal{S} \uparrow$

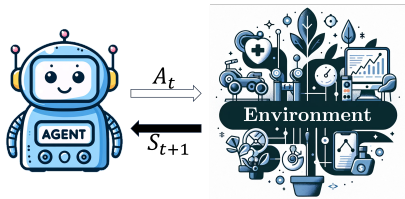


**Figure:** Real-world Environment is **Complex: Large state space**, Non-stationary dynamics, etc

Environment  $M = (\mathcal{S}, \mathcal{A}, P)$

- ▶ State  $S_{t+1} \sim P(\cdot | S_t, A_t)$ .
- ▶ Games: **Exponentially large state space** (e.g., Go  $> 10^{170}$ , Atari games  $> 128^{(160 \times 192)}$  (Raw pixels), etc.)
- ▶ Real-world applications: **High-dimensional state space** (e.g., image, video, audio, text, high-dimensional feature vectors, etc.)
  - **Healthcare: Patient state** (e.g., blood pressure, heart rate, health record ...)
  - **Chatbot (GPTs): Conversation state** (e.g., prompt, dialogue history, accessible relevant information etc.)
  - **Communication, Robotics, Agriculture**
  - ...

# Unbounded resource requirement as $\mathcal{D} \uparrow$ and $\mathcal{S} \uparrow$



**Figure:** Agent-Environment Interface.  
Experience:  $A_0, S_1, A_1, S_2, \dots$ ; and  $|\mathcal{D}| \uparrow \infty$

$\text{Agent}(\mathcal{S}, \mathcal{A}, r, \mathcal{D}_t) \rightarrow \pi_t$  to max long-term rewards

- ▶ Policy  $\pi_t = \text{Agent}(\mathcal{S}, \mathcal{A}, r, \mathcal{D}_t)$  adapted to **accumulated**  $\mathcal{D}_t$  with size  $\uparrow$  and taking **large**  $\mathcal{S}$  as input.
- ▶ Resource constraints on **memory** and **computation**.
- ▶ **NOT tractable** to **retrain the entire history** data  $\mathcal{D}$  from scratch; otherwise memory and computation requirement **growing unbounded** as  $|\mathcal{D}| \uparrow \infty$ .
- ▶ **NOT tractable** to directly handle exponentially **large**  $\mathcal{S}$ .

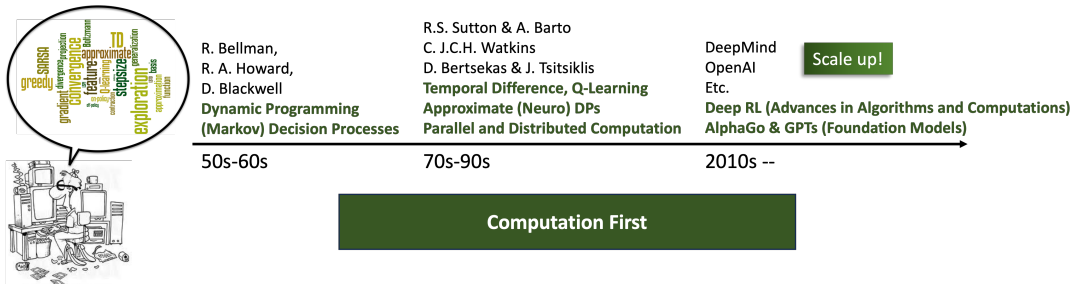


Scaling up! Then?

What's wrong with current data efficiency solutions?

Introducing HyperAgent: Simple, Efficient, Scalable  
Results

# Development of RL Algorithms: A history of "Scale up!"



- ▶ **Scale up** $\uparrow$  : (S1) **Larger** $\uparrow$  state space  $\mathcal{S}$ ; (S2) **Data**  $\mathcal{D}$  accumulated $\uparrow$  .
- ▶ **Modern RL Paradigm:** (S1) Function Approximation (Deep Neural Networks); (S2) Continuous adaptation: Incremental optimization with SGD, Experience Replay and/or Target Network.

Key for **Scalability**: (K1) **Bounded Per-step Computational Complexity**: 'NOT Scale' with  $|\mathcal{S}|$  and  $|\mathcal{D}|$ .



## Scale up $\uparrow$ AlphaGo $\rightarrow$ MuZero Series

[Silver et al., 2016, 2017, 2018, Schrittwieser et al., 2020]

- ▶  $\mathcal{S} \uparrow$ : Go  $\rightarrow$  + Board game  $\rightarrow$  + Atari.
- ▶  $\mathcal{D} \uparrow$ : human-played games (offline) + self-play (online)  
 $\rightarrow$  Purely self-play (online).

## Extremely Inefficient $\downarrow$ (e.g. AlphaGo Zero)

- ▶ **Data hungry**: 29 million ( $> 10^7$ ) games of self-play
- ▶ **Huge computation costs**: Replication would cost  $\approx$  \$35,354,222 due to data collection (sampled from simulated environment) and model computation. Training over 40 days.

# Scalability $\neq$ Efficiency: Standard Atari Benchmarks



## Scale up $\uparrow$

- ▶  $\mathcal{S} \uparrow$ : high-dimensional visual input
- ▶  $\mathcal{D} \uparrow$ : handle increasingly large amount of game-playing frames

## Inefficient $\downarrow$

- ▶ **Data:** DQN[Mnih et al., 2015] requires  $\approx 200M$  frames to reach human-level performance in Atari.
- ▶ **Deployment:** BBF [Schwarzer et al., 2023] combines  $> 15$  heuristics and tricks. Hard and laborious to tune, train and deploy.

# Scalability $\neq$ Efficiency: RLHF for LLMs

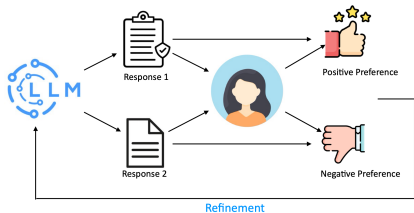


Table 4: E2E time breakdown for training a 13 billion parameter ChatGPT model via DeepSpeed-Chat on a single DGX node with 8 NVIDIA A100-40G GPUs.

Model Sizes	Step 1	Step 2	Step 3	Total
Actor: OPT-13B, Reward: OPT-350M	2.5hr	0.25hr	10.8hr	13.6hr

Table 5: E2E time breakdown for training a 66 billion parameter ChatGPT model via DeepSpeed-Chat on 8 DGX nodes with 8 NVIDIA A100-80G GPUs/node.

Model Sizes	Step 1	Step 2	Step 3	Total
Actor: OPT-66B, Reward: OPT-350M	82 mins	5 mins	7.5hr	9hr

## Scale up $\uparrow$

- $\mathcal{S} \uparrow$ : more complex, diverse or longer conversations
- $\mathcal{D} \uparrow$ : incrementally adapt to extensive online human feedbacks

## Inefficient $\downarrow$

- **Data:** Human feedback is scarce and expensive in alignment problem. (1.5M (Offline) and 1.7M (Online) in LLaMA2 [Touvron et al., 2023])
- **Computation:** RLHF occupies most of the training time. [Yao et al., 2023]

# Efficiency Challenges in Modern RL: Summary

Modern RL is **scalable**  $\uparrow$ , much success in simulated environment.

- ▶ **Modern RL Paradigm:** (S1) DNN; (S2) Incremental optimization
- ▶  $\Rightarrow$  **Scalable** Algorithm to handle (S1)  $\mathcal{S} \uparrow$ ; and (S2)  $\mathcal{D} \uparrow$  with (K1) **Bounded Per-step Computational Complexity**.

Modern RL is **inefficient**  $\downarrow$ , an obstacle for **real-world applications**.

- ▶ **(E1) Data Hungry:** Collecting data can be expensive and time-consuming in real-world.
- ▶ **(E2) Computation:** The per-step computation cost, although bounded (K1), is still high since Increasingly larger deep network, e.g. AlphaGo ( $> 30M$ ), GPT-3.5 (175B) and GPT-4 ( $> 1T$ ).
- ▶ **(E3) Deployment:** Many heuristic training tricks and complicated components. Laborious to tune and deploy. Engineering cost is high, especially for **real-world applications**.

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# Research question?

Towards fulfilling the promise of RL in real-world complex environment, can we design

- ▶ (A1) **Simple** Algorithm easy to use and deploy (E3)
- ▶ (A2) **Efficient** Algorithm low data (E1) and computation cost (E2)
- ▶ (A3) **Scalable** Algorithm large  $\mathcal{S} \uparrow$  (S1) and accumulated  $\mathcal{D} \uparrow$  (S2)

“To complicate is easy. To simplify is difficult.”

– Bruno Munari



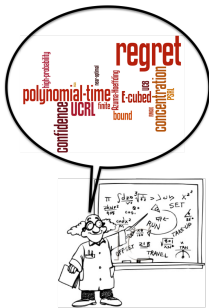
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## Data Efficiency under Function Approximation: Theoretical Effort



R.A. Fisher, William R. Thompson (30s)  
T. L. Lai and H. Robbins (80s)  
**Sequential design and allocations (MAB)**

Before 90s

E3/PSRL/UCRL ...  
**Tabular RL**

2000 --

- Eluder dimension/Information Ratio
- Bellman Rank/Bilinear Structure
- Decoupling coefficient/DEC/GEC ...
- Structural assumption and Algorithms for RL with function approximation**

2014 --

## Data First

- ▶ **X Intractable computation:** intricate nonconvex optimization [Jiang et al., 2017, Jin et al., 2021, Du et al., 2021, Foster et al., 2021, Liu et al., 2023] or sampling from intricate distribution [Zhang, 2022, Dann et al., 2021, Zhong et al., 2022].
- ▶ **X Unbounded memory and computation:** e.g. need to re-train entire history for each episode (with regression oracle) [Osband et al., 2019, Wang et al., 2020, Ishfaq et al., 2021, Agarwal et al., 2023]


## What's wrong with current data efficiency solutions?

# Data Efficiency in Deep RL: Practical Work

Algorithm	Components
DDQN	incremental SGD with experience replay and target network
Rainbow	(DDQN) + Prioritized replay, Dueling networks, Distributional RL, Noisy Nets.
BBF(23)	(DDQN) + Prioritized replay, Dueling networks, Distributional RL, Self-Prediction, Harder resets, Larger network, Annealing hyper-parameters.

**Table:** The extra techniques used in different algorithms, e.g. DDQN [Van Hasselt et al., 2016], Rainbow [Hessel et al., 2018], BBF [Schwarzer et al., 2023].

- ▶ **✓ Scalable:** e.g. DDQN use incremental SGD with experience replay and target network.
- ▶ **✗ Not Simple:** Complicated component and many heuristic tricks. Hard and laborious to tune.
- ▶ **✗ Not Efficient: Provably inefficient:** e.g. BBF use  $\epsilon$ -greedy which need **exponential many sample in some environment, provably** [Kakade, 2003, Strehl, 2007, Osband et al., 2019, Dann et al., 2022].  
**Practically inefficient:** **Per-step computational cost is high**, e.g. BBF uses larger networks.



**Theory! Data First**

regret  
polynomial-time  
confidence  
UCRL  
finite  
bound  
concentration  
PSRL  
UCB  
t-cubed  
RMSE

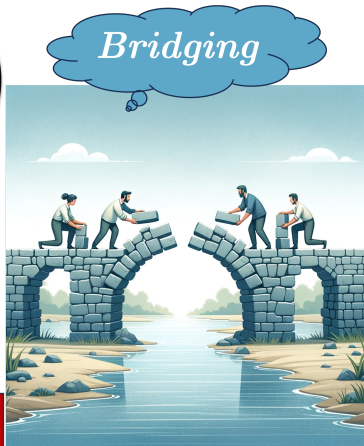



**Scale up! Computation First**

greedy  
SARSA  
divergence  
projection  
convergence  
feature  
Boltzmann  
approximate  
TD  
generalization  
exploration  
stepsizes  
Q-learning  
on-policy  
off-policy  
contraction  
basis  
approximation  
function

**regret**  
 polynomial-time  
 confidence  
 UCRL  
 concentration  
 high-probability  
 new-optimal  
 finite  
 UCB  
 t-cubed  
 bound  
 PSRL

**Theory! Data First**



**greedy**  
 Sarsa  
 TD  
 convergence  
 feature  
 exploration  
 approximate  
 stepsize  
 generalization  
 basis  
 approximation  
 function  
 gradient  
 divergence  
 projection  
 Boltzmann  
 on-policy  
 off-policy  
 contraction

**Scale up! Computation First**

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# HyperAgent: Simple and Scalable Algorithmic Component

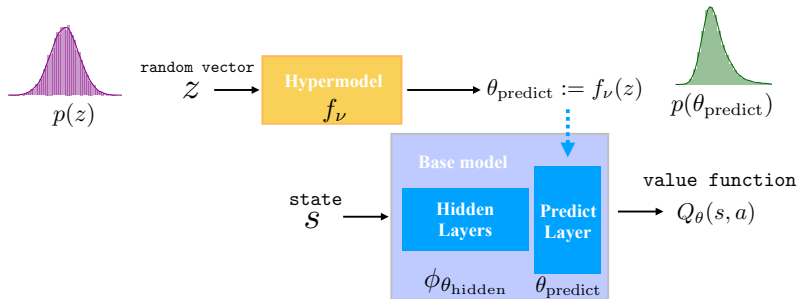
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<b>HyperAgent</b>	<b>Hypermodel</b>

**Table:** The extra techniques used in different algorithms, e.g. DDQN [Van Hasselt et al., 2016], Rainbow [Hessel et al., 2018], BBF [Schwarzer et al., 2023] and **our HyperAgent**.

- ▶ **✓ Simple:** Compared to DDQN [Van Hasselt et al., 2016], only one additional component, hypermodel, that is easily compatible with all Feedforward Deep Networks.
- ▶ **✓ Scalable:** Incremental SGD under DNN function approximation, same as DDQN.

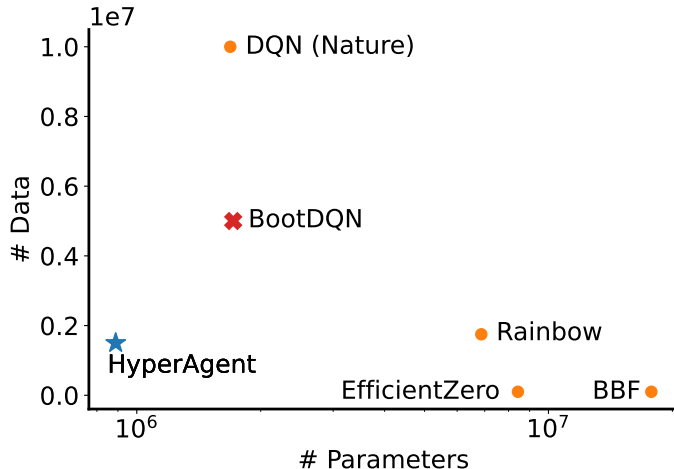
# HyperAgent: Hypermodel

- ▶ Base model: DQN-type structure  $Q_{\theta}(s, a) = \langle \phi_{\theta_{\text{hidden}}}(s), \theta_{\text{predict}}(a) \rangle$ .
- ▶ Hypermodel:  $\theta_{\text{predict}} = f_{\nu}(z)$  where  $z \sim p(z)$ .  $p(z)$  is a fixed reference distribution.



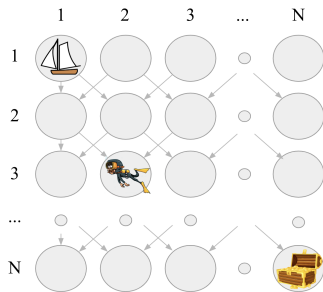
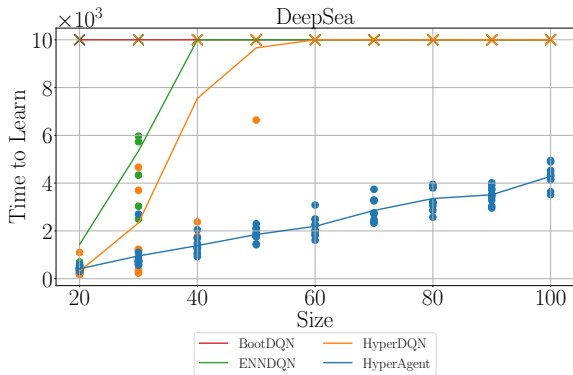
Resulting model:  $Q_{\theta_{\text{hidden}}, f_{\nu}(z)}(s, a)$  is a randomized value function depends on  $(s, a)$  and additional random variable  $z$ .

# HyperAgent: Efficiency in benchmarks (Atari)



- ▶ **✓ Data efficient:** 15% data consumption of DQN[Mnih et al., 2015] by Deepmind.
- ▶ **✓ Computation efficient:** 5% model parameters of BBF[Schwarzer et al., 2023] by Deepmind.

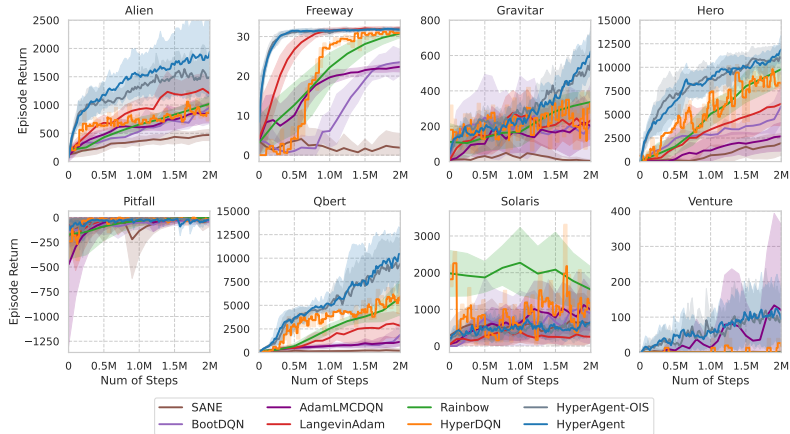
# HyperAgent: Efficiency in benchmarks (DeepSea)



**Figure:** Comparative results on DeepSea with BootDQN [Osband et al., 2018], HyperDQN [Li et al., 2022], ENN-DQN[Osband et al., 2023]. The y-axis represents the number of episodes required to learn the optimal policy for a specific problem size. The symbol  $\times$  indicates the algorithm was unable to learn within  $10^4$  episodes.

► ✓ scalable as size  $\uparrow$ . ✓ data efficient: optimal episode complexity is **linear in the size** of the problem.

# HyperAgent: Efficiency in 8 hard exploration tasks



**Figure:** Comparative results on 8 hardest exploration games. HyperAgent shows stable performance and exploration efficiency compared with randomized RL algorithm including other approximate posterior sampling methods.

# HyperAgent: Theoretical Guarantees in RL

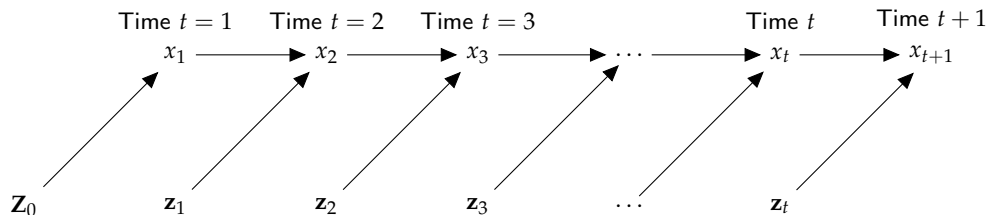
Algorithm	Practical General FA			Theoretical Finite-horizon Tabular	
	FA	Incremental	Efficiency	Regret	Per-step computation
PSRL[Osband and Van Roy, 2017]	✗	✗	✗	$H^2\sqrt{SAK}$	✓ $S^2A$
RLSVI[Osband et al., 2019]	✓	✗	✗	$H^2\sqrt{SAK}$	✓ $S^2A$
Ensemble+[Osband et al., 2019]	✓	✓	●	N/A	N/A
Bayes-UCBVI[Tiapkin et al., 2022]	✗	✗	✗	$\sqrt{H^3SAK}$	✓ $S^2A$
Incre-Bayes-UCBVI[Tiapkin et al., 2022]	✓	✓	●	N/A	N/A
LMC-LSVI[Ishfaq et al., 2023]	✓	✓	●	$H^2\sqrt{S^3A^3K}$	✗ $K \cdot S^2A \cdot \log SAHK$
HyperAgent	✓	✓	✓	$H^2\sqrt{SAK}$	✓ $S^2A \cdot \log SAHK$

- ▶ Finite-horizon tabular: # states:  $S$ , # actions:  $A$ , horizons:  $H$ , # episodes:  $K$
- ▶ PSRL and Bayes-UCBVI requires dirichlet prior over transitions, otherwise **computation intractable**; RLSVI requires gaussian noise, otherwise **unbounded per-step computation  $\tilde{O}(K)$** .
- ▶ The lemma 3 in [Osband and Van Roy, 2017] target for time-homogeneous MDP may not be correct as pointed out in [Qian et al., 2020]. By a careful revisit, the bound can be corrected to  $H^2\sqrt{SAK}$  for time-inhomogeneous setting.

# HyperAgent: Possible theoertical extensions

- ▶ We already have a theoretical results in linear bandit, which RL with  $S = 1, H = 1$  and linear function approximation.
- ▶ Immediat extension to RL under Linear Function Approximation ( $H > 1$ ) pose no much more difficulty.
- ▶ Extension to infinite horizon average-reward RL is doable. I have some preliminary results.
- ▶ Extension to function approximation with generalized linear model and neural tangent kernel is possible.

# The novelty and difficulty in the mathematical analysis: No Prior Art

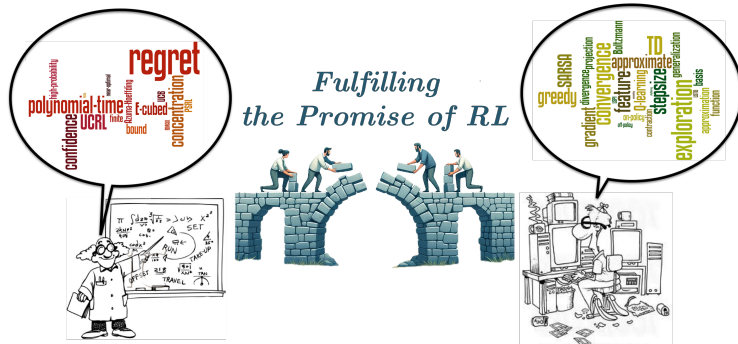


**First probability tool for sequential random projection.** A **Non-trivial** martingale extension of the Johnson–Lindenstrauss lemma and Subspace embedding.

- ▶ **Difficulty:** **Sequential dependence** of **high-dimensional** R.V. due to the adaptive nature of Sequential Decision Making.
- ▶ **Novelty:** A novel and careful **construction of stopped process** with **non-trivial application of 'method of mixtures'** in self-normalized martingale.



# Simple, Efficient, Scalable: Bridging Theory and Practice



HyperAgent is the **first** principled RL agent that is

- ▶ **Simple, Efficient** and **Scalable**;
- ▶ **Empirically** and **Theoretically** justified. **No Prior Art**.

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